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RAINDROP DISTRIBUTIONS NEAR FLAGSTAFF, ARIZONA

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UNIVERSITY OF ILLINOIS

for

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Durham, North Carolina

CONTENTS

ILLUSTRATIONS

TABLES

ABSTRACT

The Illinois State Water Survey raindrop camera was operated from 9 July through 19 August 1967 at the Fort Valley Experimental Forest near Flagstaff, Arizona, before, during, and after a period of cloud seeding. Drop-size distributions were determined for showers which were seeded and unseeded and showers with and without the occurrence of hail. It was found, on the average, that seeded showers and natural showers forming in more humid air masses have fewer large drops and more drops per cubic meter for the same rainfall rate than have showers accompanied by hail and unseeded showers forming in relatively dry air masses. The increase in the relative number of drops at rainfall rates greater than 30 mm hr^{-1} was investigated and found to be the result of the breakup of drops larger than 2.0 mm diameter.

It is concluded that the peculiarity of the Flagstaff distributions lies in the extraordinary number of very large drops which fall in showers forming in relatively dry air and that these large drops do not break up easily because they are the remnants of hailstones.

RAINDROP DISTRIBUTIONS NEAR FLAGSTAFF, ARIZONA

by

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INTRODUCTION

As described by Fujita et al. (1962) the area around the San Francisco Peaks near Flagstaff, Arizona, has been found to be a convenient atmospheric laboratory for the study of orographic cumulonimbus during the summer monsoon. Hardy (1963) collected raindrop-size measurements south-southwest of the Peaks in 1961 using the measurements as illustrations of the roles played by accretion, coalescence, and evaporation in the formation of a raindrop-size distribution. A modified raindrop camera of the Illinois State Water Survey (Jones, 1959; Stout and Mueller, 1968) collected data on the drop-size distribution during the summer shower seasons of 1963, 1966, and 1967. Collection of raindrop-size data in 1967 was in conjunction with studies of the physics of the clouds in the Flagstaff area by Meteorology Research, Inc., through the support of the U. S. Bureau of Reclamation. Artificial nucleation of the clouds was a primary tool in their study. Thus, raindrop data could be collected from untreated and artificially nucleated clouds for comparison of the two. The collection period was arranged to begin before cloud seeding was started, to continue through the seeding period, and to end some time after the effects of the cloud seeding had disappeared.

The first data were collected by the camera on 9 July and the last on 19 August. Cloud seeding was performed on most of the days between 18 July and 10 August as summarized in Table 1. All cloud seeding was performed by aircraft 150-300 m below cloud base to take advantage of the updraft into the cloud. The type of seeding classified as "Isolated" means that an isolated cloud was seeded by circling underneath it for 20-25 minutes. The line seeding was performed by flying along a line of some length for the time interval shown. Although Meteorology Research, Inc., attempted to select clouds that could rain over the drop measuring site, this was not always possible within the limits of their seeding criteria and many clouds were seeded at sites away from the drop camera.

OBSERVATIONS

The raindrop camera was installed in an open area of the Fort Valley Experimental Forest (FTV) approximately 11 km north-west of Flagstaff, Arizona. An anemograph, recording raingage, and Stevenson screen were erected close to the camera. The screen contained maximum and minimum thermometers and a hygrothermograph with a 29-hr chart drive. Intermittent pilot balloon observations were made from a meadow approximately 500 m south of the camera.

Table 1. Seeding Operations, 1967

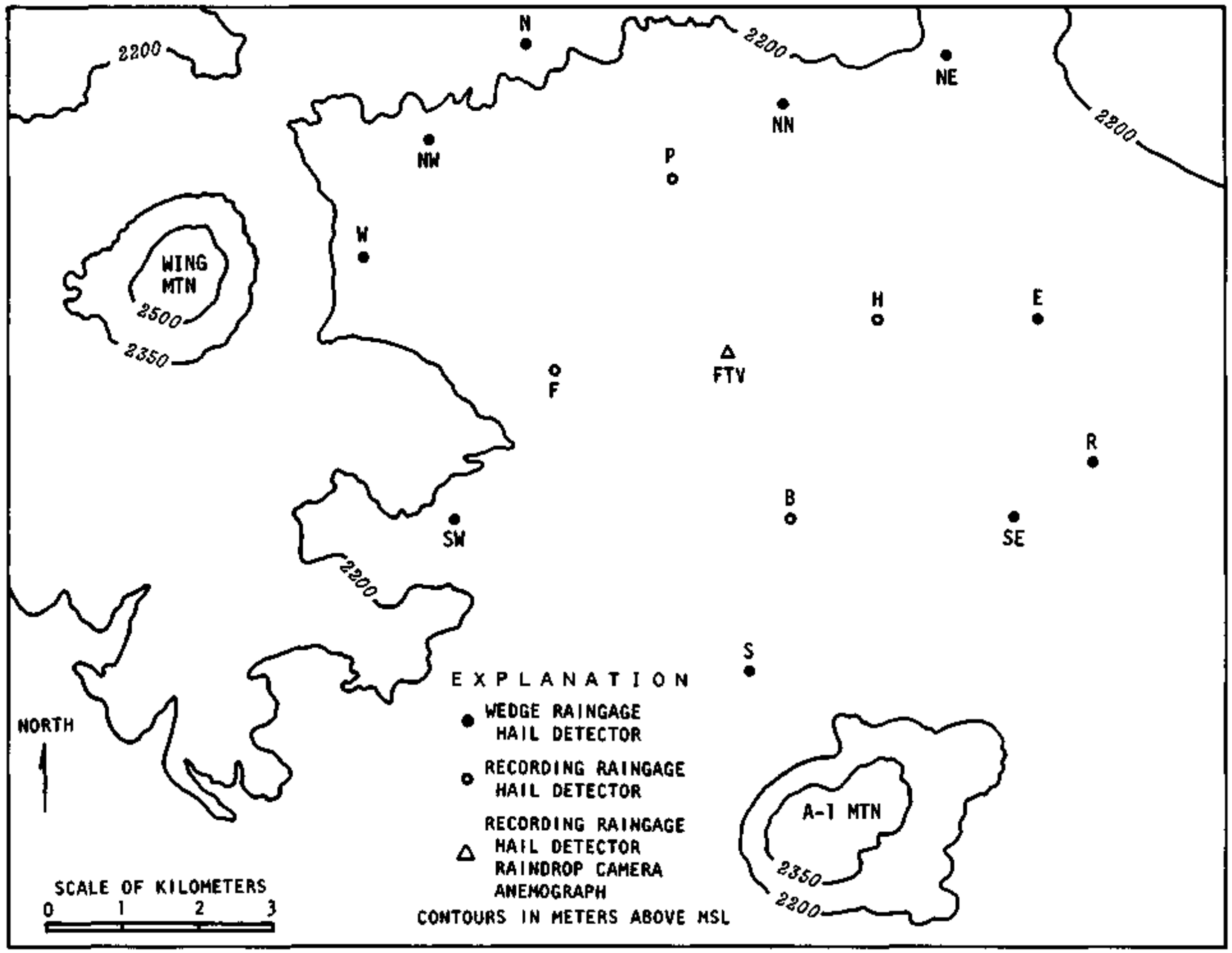
Date	Time, MDT	Type
7/18	1518-	Isolated
7/19	1045-	Isolated
7/21	1310-	Isolated
	1345-1425	Line
7/24	1033-	Isolated
7/25	1250-	Isolated
7/27	1152-	Isolated
	1224-1340	Line
7/29	1102-	Isolated
7/31	1322-	Isolated
	1400-1452	Line
8/01	1155-1405	Line
	1512-1646	Line
8/02	1129-	Isolated
8/03	1144-	Isolated
8/05	1149-1248	Line
8/08	1234-	Isolated
8/09	1225-1322	Line
8/10	1117-	Isolated

In addition to the recording instruments located near the camera, four recording raingages were operated in a circle of radius 1.2 km with the camera as the center. All recording raingages were fitted with 6-hr chart drives and 321-mm diameter tops to permit the reading of 2-minute accumulations of 0.1-mm accuracy with the weighing-bucket type gages. Eight plastic wedge raingages were placed as evenly as the terrain would permit on a radius of approximately 2.4 km about the camera. Each raingage site included a passive hail indicator of the type described by Schleusener and Jennings (1960) and Wilk (1961). These hail indicators are made of 30 cm by 30 cm styrofoam plastic, 2.5 cm thick, covered with household aluminum foil. The indicator was suspended above the ground by wire legs stuck into the sides of the plastic. The impact of a hailstone upon the indicator leaves a characteristic imprint in the foil covering. The imprint size is related to the size of the hailstone causing it. The network and the surrounding terrain are shown in Pig. 1. It will be noted that the cinder cones of Wing Mountain and A-1 Mountain were 4 km west and 3.5 km south, respectively, of the camera site.

RAINDROP CONCENTRATIONS

Mueller (1967) found from the 1966 raindrop camera data from Flagstaff that the drop-size distributions could be separated into two categories: 1) a high concentration of drops per unit volume per rainfall rate similar to distributions measured in more humid climates of the world; and 2) a low concentration which has been observed in the dry climate of the southwest United States. There was a paucity of raindrops smaller than 1.5-mm equivalent spherical diameter in the low concentration rains and, necessarily, an excess of larger drops for a given rainfall rate as compared with the high concentration rains.

The small number of storm days (8) photographed in 1966 precluded a definitive investigation of the physical mechanisms responsible for the formation of the two concentrations. It was noted that the low concentration rains were usually accompanied by small hail, and upper air soundings showed that, on the average, lower relative humidities were found at the 700-mb


N
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SW
SE
2200
S
EXPLANATION
WEDGE RAINGAGE
HAIL DETECTOR
RECORDING RAINGAGE
HAIL DETECTOR
RECORDING RAINGAGE
HAIL DETECTOR
RAINDROP CAMERA
ANEMOGRAPH
CONTOURS IN METERS ABOVE MSL
NORTH
SCALE OF KILOMETERS
0 1 2 3
A-1 MTN
2350
2200

Fig. 1

level for low concentration days than were found on days of high concentration rains. The lower relative humidities aloft on days of low concentration rains would qualitatively indicate that evaporation below cloud base would be responsible for the loss of the smaller drops. Quantitatively, it was noted that evaporation, on the average, could not explain all of the difference between the high and low concentrations. Mueller also noted that silver iodide (AgI) crystals could have entered the storms having high concentrations on 3 of the 5 days when Meteorology Research, Inc., was seeding in the area. Since cloud-seeding materials are injected into the clouds to increase the number of available nuclei and should increase the number of precipitating drops, cloud seeding may explain the observance of high raindrop concentrations in the dry atmosphere of north-central Arizona.

Table 2 is a summary of the 1966 drop-size observations.

Table 2. 1966 Flagstaff Raindrop Samples

Date	Max. rate (mm/hr^{-1})	No. Samples (m^{-3})	Hail	Seeded	Conc. type
7/18	19	20	No	Yes	Low
7/21	77	101	No	Yes	High
7/25	2	21	No	Yes	High
7/27	49	42	No	Yes	High
7/29	9	61	No	Yes	Low
8/02	63	36	No	No	Low
8/08	37	61	Yes	No	Low
8/10	59	100	Yes	No	Low

The data collection procedure in 1967 was intentionally designed to study the phenomena of the high and low concentration rains and the conditions under which they occurred. The hail indicators detected the occurrence of hail within a radius of 2.4 km of the camera; the recording raingage records defined the position of the camera with respect to the center of the rain cell; pibals were taken to determine the windflow over the camera; and special radiosonde releases were made 19 km southwest of the camera site at the time most likely to correspond to the occurrence of active convection near the camera site. Meteorology Research, Inc., cooperated within the limits of the seeding criteria by choosing clouds over the camera site for their seeding experiments.

Normal precipitation accumulation at Port Valley for July and August is 130 mm, and 221 mm fell during the 40 days that the camera was installed.

Definition of Concentrations

A high concentration rain was defined as one which had a coefficient, A, equal to or greater than 23 in the equation $N_T = AR^b$ relating the total number of drops per cubic meter to the rainfall rate, R, expressed in millimeters per hour. A low concentration rain was defined as a rain having a coefficient of 22 or less. Unlike the 1966 rains, the 1967 rains exhibited a continuum of coefficients from 8 through 48. A coefficient of 23 was selected as the division criterion for both years because most of the seeded rains had coefficients larger than 23, whereas most of the showers with hail had coefficients smaller than 23. Table 3 summarizes the data collected in 1967. The increase in rainfall and in the data collected in 1967 over 1966 also complicated the data analysis by providing more complex shower situations and more than one shower on many of the days. In Table 3 are shown the date and time of the rain, the number of cubic meter samples, the concentration type, and the maximum rainfall rate (R max.), as determined from the drop-size data.

Fig. 2 shows the average curve for N_T-R for the high concentration showers compared with the average curve of the same quantities for the low concentration showers.

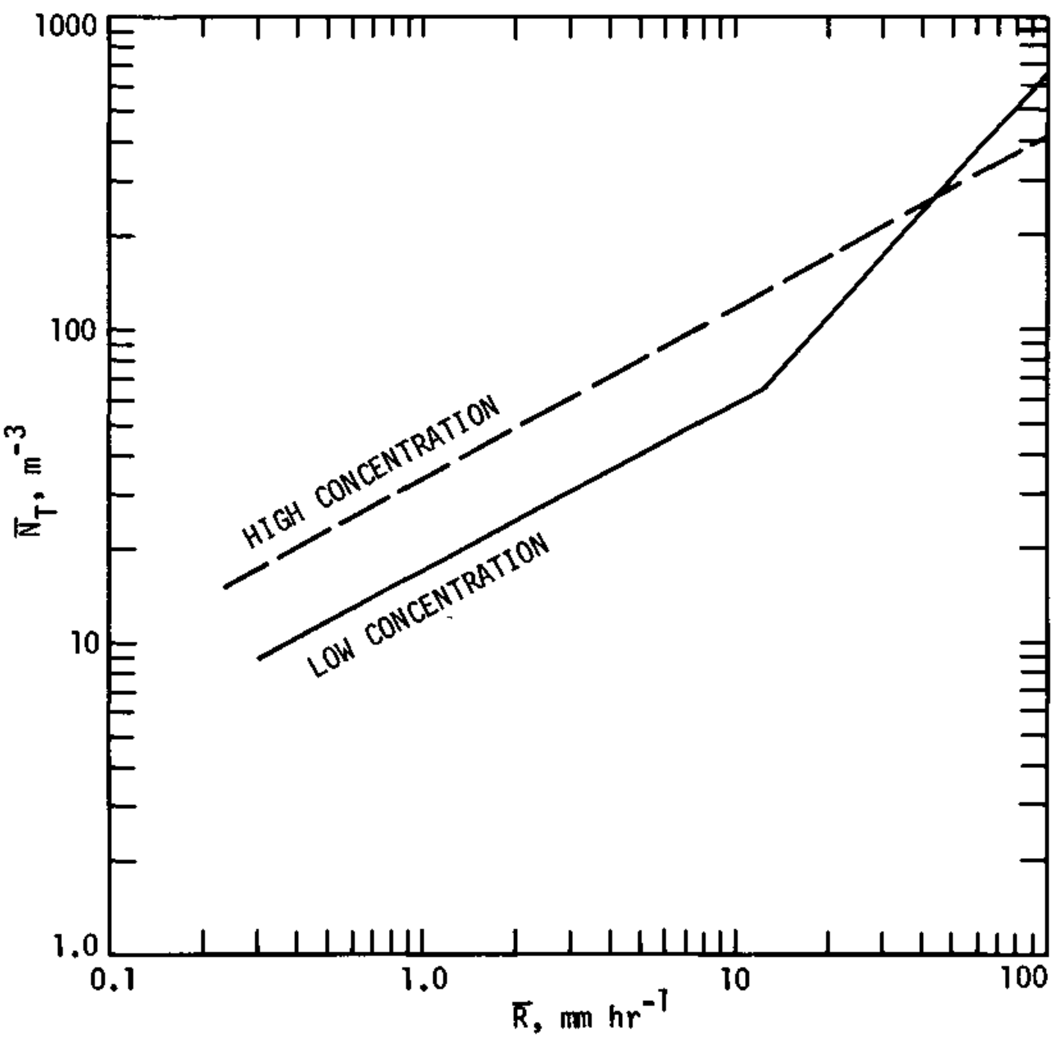
<u>Time-Shift of Raindrop Concentrations</u>

Both observation and theory show that the skewness of drop-size distributions as plotted in Fig. 2 changes during a shower. The distribution changes from a relatively large number of big drops at the beginning of a shower toward a preponderance of small drops and no large drops at the end of the shower. Fig. 3 is an illustration of the shift with time of the number of drops per cubic meter in the storm of 3 August 1967. Since the shower did not fall at a constant rainfall rate of 1.0 mm hr , the data have been normalized by assuming that b = 1.0 in the relationship, $N_T = AR^b$. The actual values of N_T and R for the storm of 3 August 1967 were applied in the equation to determine the value of A.

Table 3. Summary of 1967 Observations

Date	Time, MDT	Number of m^3 samples	Conc. type	R max., mm hr^{-1}
7/09	1653-2042	40	High	1.0
7/10	1439-1455	15	Low**	19.0
7/10	1530-1638	54	High	9.1
7/12	1617-1916	75	Low**	153.6
7/13	1825-1837	13	Low	39.1
7/13	1910-1942	32	High	35.0
7/13	1957-2041	46	High	17.6
7/14	1400-2000	24	Low	2.1
7/15	1602-1714	69	Low**	114.6
7/15	1813-1818	6	Low**	18.0
7/16	1424-1451	28	Low	52.0
7/16	1556-1605	7	Low	13.5
7/17	1733-1743	11	Low	34.2
7/26	1140-1234	54	Low**	59.9
7/27	1418-1524	73	High*	3.1
7/29	1203-1454	90	High*	40.4
7/30	1303-1615	147	High	3.4
8/01	1540-1650	56	High*	33.9
8/03	1225-1245	21	Low**	92.3
8/03	1302-1311	10	Low	6.9
8/03	1333-1342	10	High*	3.8
8/04	1707-1746	34	High*	2.4
8/06	0013-0023	5	Low	1.3
8/06	1738-1756	18	High	14.0
8/06	1909-1930	18	High	3.5
8/06	2105-2124	18	Low	8.7
8/07	0819-1112	48	High	10.8
8/07	1235-1249	15	High	11.6
8/08	1507-1618	84	High	14.2
8/09	1253-1304	11	Low	8.0
8/09	1313-1343	30	High**	25.4
8/09	1411-1431	19	High	1.5
8/10	1406-1414	9	Low	2.8
8/10	1424-1709	39	High	2.0
8/14	1543-1602	13	Low	8.1
8/15	0044-0049	6	High	5.5
8/15	1306-1328	22	Low**	41.0
8/15	1427-1450	24	Low**	29.8
8/15	1451-1456	6	High	6.2
8/15	1457-1525	29	Low**	90.3
8/15	1526-1542	17	High	3.4
8/15	1552-1611	20	Low	25.6
8/15	1612-1640	29	Low	6.0
8/16	1541-1550	11	High	22.7
8/16	1813-1821	6	High	3.3
8/18	1256-1306	14	Low**	8.7
8/18	1350-1401	12	Low	7.9
8/18	1402-1434	48	High	12.9
8/19	1530-1605	28	Low**	17.2

* Seeded Storm ** Hail Observed


1000
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1.0
0.1
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10
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$\overline{N}_T$, m^{-3}
$\overline{R}$, mm hr^{-1}
HIGH CONCENTRATION
LOW CONCENTRATION

Fig. 2

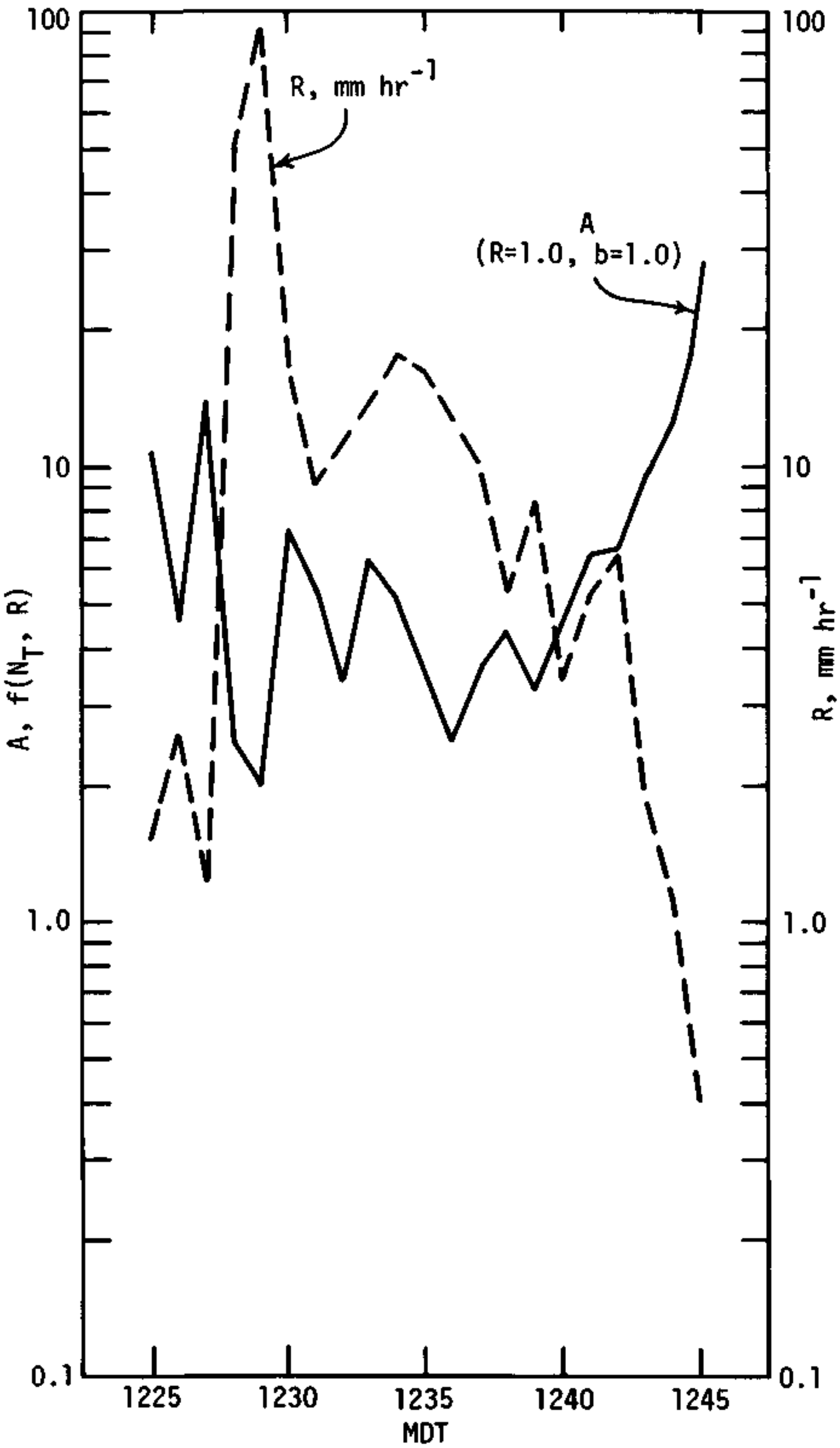

100
10
1.0
0.1
A, f(N_T, R)
R, mm hr^-1
A
(R=1.0, b=1.0)
1225
1230
1235
1240
1245
MDT

Fig. 3

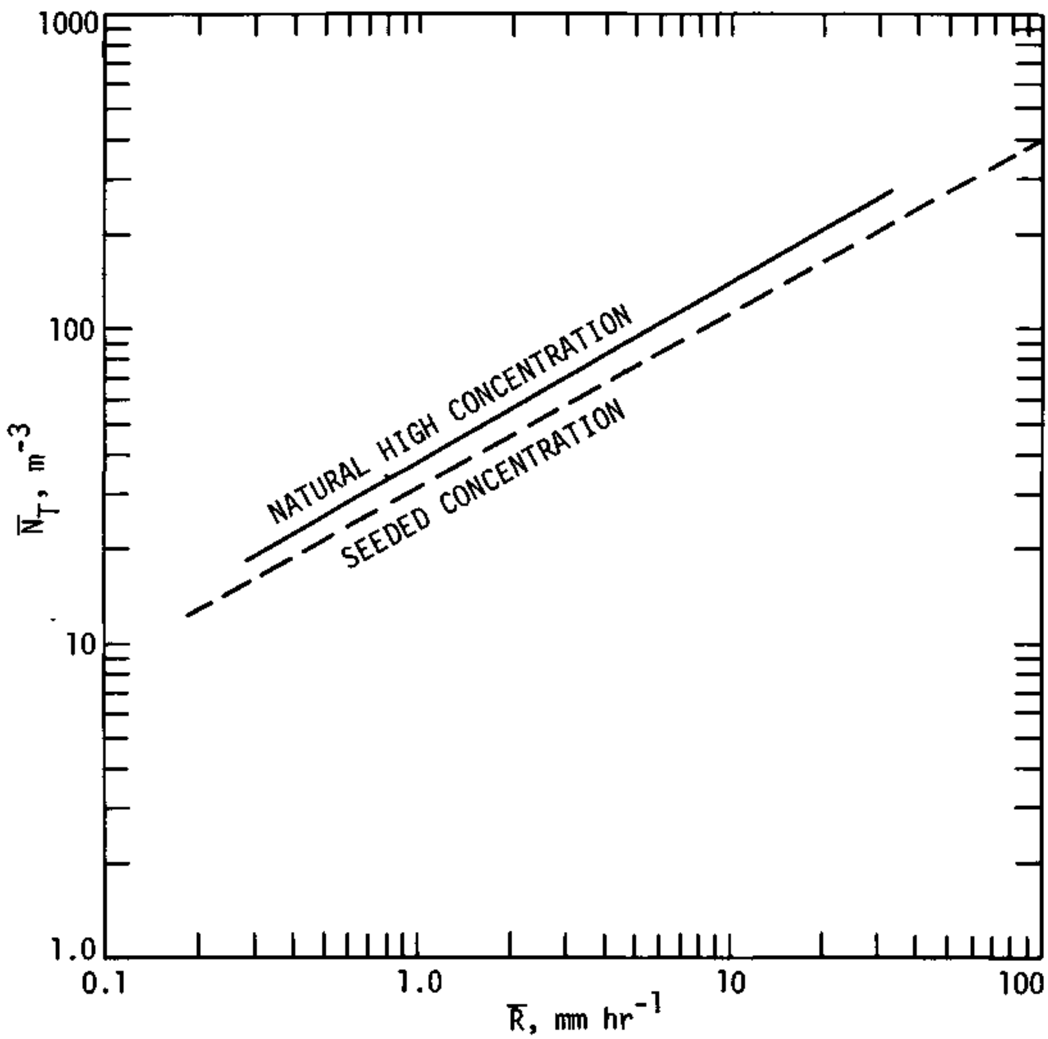
The resultant values of A are plotted in Fig. 3 against the time of the observation. The rainfall rate is plotted also in this figure.

This shift with time of relative distribution of drops per cubic meter is attributed to the difference in fall velocity between large and small drops (Atlas and Plank, 1953)• Assuming that the drops leave the cloud-generating level at the same time, the larger drops, which have the highest fall velocity, will reach the surface before the smaller drops, resulting in the formation of a low concentration rain at the beginning of the shower and a shift to high concentration as the shower progresses and comes to an end. For a fixed rainfall rate, the number of drops must increase as the drop-size decreases.

It might be argued that this normal tendency can account for the observations near Flagstaff during the summers of 1966 and 1967 through a mechanism of selective sampling by chance. This argument was rejected since it was found that the radar echoes associated with the showers indicated that the centers of the showers passed close to the sample site. Most of the showers were composed of more than one cell (Byers and Braham, 1949) and had been producing rain before rain fell at the camera.

Seeded Concentrations

Fig. 4 shows the N_T-R relationships for the average natural high concentration shower and the average artificially seeded shower. Natural seeding from sea salt nuclei, soil particles, other atmospheric particulants, and ice crystals from the tops of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10
1.0
0.1
1.0
10
100
$\bar{N}_T$, m^{-3}
$\bar{R}$, mm hr^{-1}
NATURAL HIGH CONCENTRATION
SEEDED CONCENTRATION

Fig. 4

earlier storms are presumed to have resulted in the high concentration showers falling through a relatively humid air mass. Hail was found to have occurred with only one of these storms (8/9/67, Table 3). Artificial seeding of aerosol particles from both wood smoke particles* and AgI was found to result in concentrations very similar to the natural high concentrations (Table 3). The wood smoke was probably activated by gasoline engine exhaust from the nearby US Route 66 in the process suggested by Schaefer (1968).

The natural average concentration and the seeded concentration curves are displaced by 7 drops per cubic meter at R = 1 mm hr^{-1} and are essentially parallel. If the difference is not sampling error, the inference is that natural processes are more efficient at producing drops than is AgI seeding as performed in 1967.

Hail Concentrations

Fig. 5 is a plot of the curves for the average hail and low concentration cases. The two curves are separated by 8 drops per cubic meter at R = 1 mm hr^{-1} and both fall within the criterion of having an A < 23. The peculiar difference between the two curves lies in the very definite nodal point between 30 and 40 mm hr~ for the hail concentration. On the basis of a suggestion by Styber (1961), it was thought that all low concentration showers probably contained hail even though it was not detected. The

*Smoke particles from the burning of uprooted Juniper trees 24 km east of Port Valley probably acted as ice nuclei for the storm of 8/4/67 as determined by low-level wind trajectories.

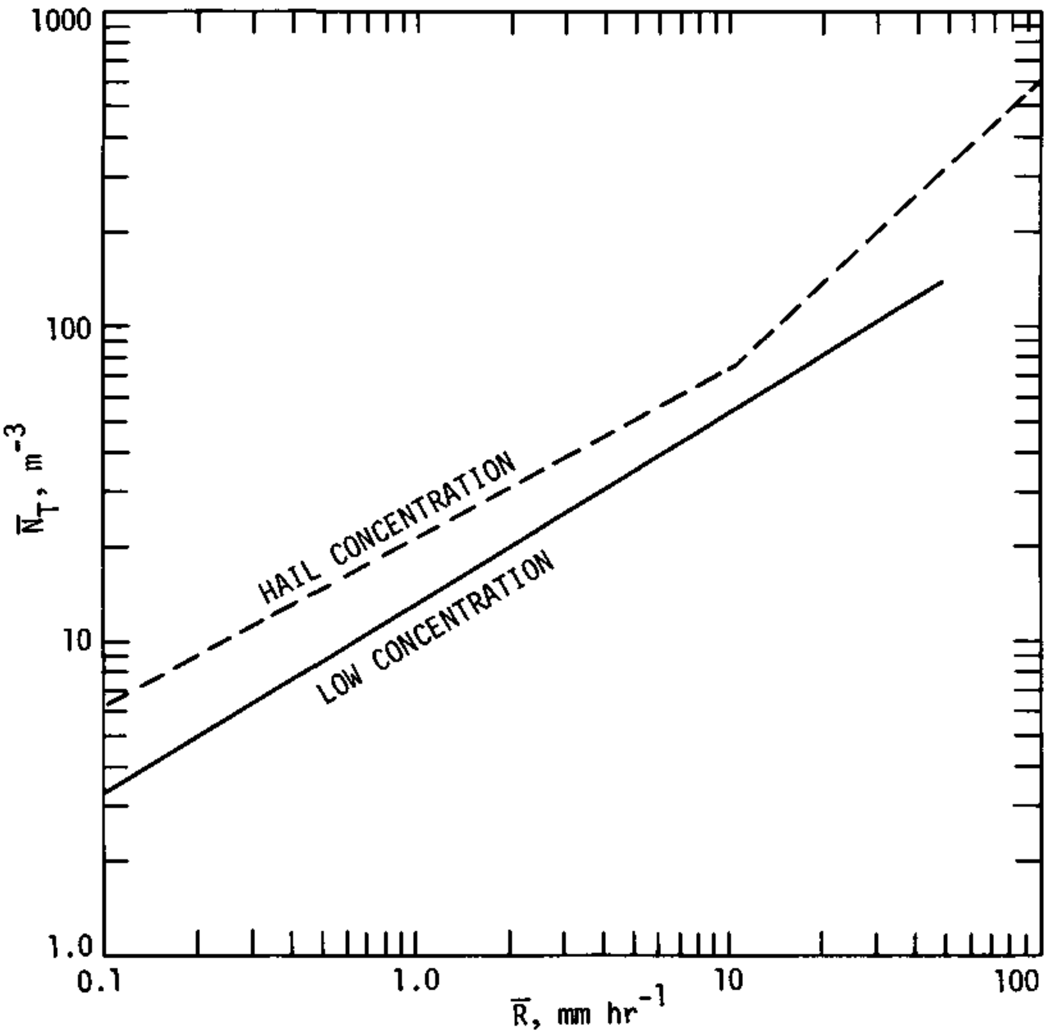

1000
100
10
1.0
0.1
1.0
10
100
$\overline{N}_T$, m^{-3}
$\overline{R}$, mm hr^{-1}
HAIL CONCENTRATION
LOW CONCENTRATION

Fig. 5

difference between the curves, particularly the change in slope of the hail curve, makes this suggestion unlikely; at least the possibility of hail having been in existence close to the earth's surface in the low concentration cases is unlikely. However, there were only three observations of rainfall rates over 30 mm hr^{-1} for the low concentration showers without hail, and no good trend of a regression line can be determined from these three points.

The air masses in which low concentration showers occur have been found to be consistently drier than the high concentration air masses. It has also been found that hail, with one exception, occurred only with low concentration showers. It is hypothesized that an unstable, relatively dry air mass is necessary for the formation of hail within a shower. The function of the drier air is to provide the heat sink, through evaporation from the raindrops, necessary to remove the heat of glaciation released as the raindrops freeze. Also, the freezing must occur at temperatures only a little colder than O C since colder temperatures would remove the heat of glaciation without the aid of evaporative cooling. Freezing nuclei active at relatively warm temperatures must be present, but in limited numbers since large numbers of nuclei would encourage the formation of many ice particles and result in a high concentration shower of many small drops. These high concentration, high altitude showers may be the source of the virga often observed over the desert.

Support for the above hypothesis may be found in Table 4. It will be noted in Table 4 that the largest drop found in a high concentration shower was of D = 6.0 mm (7/29/67), whereas the largest diameter found in a low concentration shower was 7.3 mm (7/16/67) and the largest diameter in a hail shower was 7.9 mm (7/12/67, 7/26/67, and 8/3/67). A number of investigators (Cotton and Gokhale, 1967; Blanchard, 1950; Gunn and Kinzer, 1949) have stated that the largest drop diameter to be expected in rain is somewhat less than 6.0 mm. Above 6.0 mm diameter the raindrops become sufficiently unstable in falling through the air to break up into smaller drops. However, Blanchard (1950) has found that the presence of an air bubble within the water drop will stabilize the drop so that it may continue to grow without breakup to a much larger size. Since the showers accompanied by hail and the low concentration showers were found to have drops of diameter larger than 6.0 mm, the most likely explanation for the presence of these oversized drops is that the drops either contain a small ice particle or, more likely, contain an air bubble. This bubble would have been entrained in the hailstone which upon melting resulted in the raindrop. The oversized raindrops when observed were not unmelted hailstones as evidenced by the forward scattering of the lighting used to photograph the drops in silhouette. Hailstones which have been photographed by the camera do not show sufficient forward scatter of visible light to be detected on the photographic film.

Table 4. Concentration Types, the Occurrence of Hail, and Maximum Drop Diameter in Each Storm

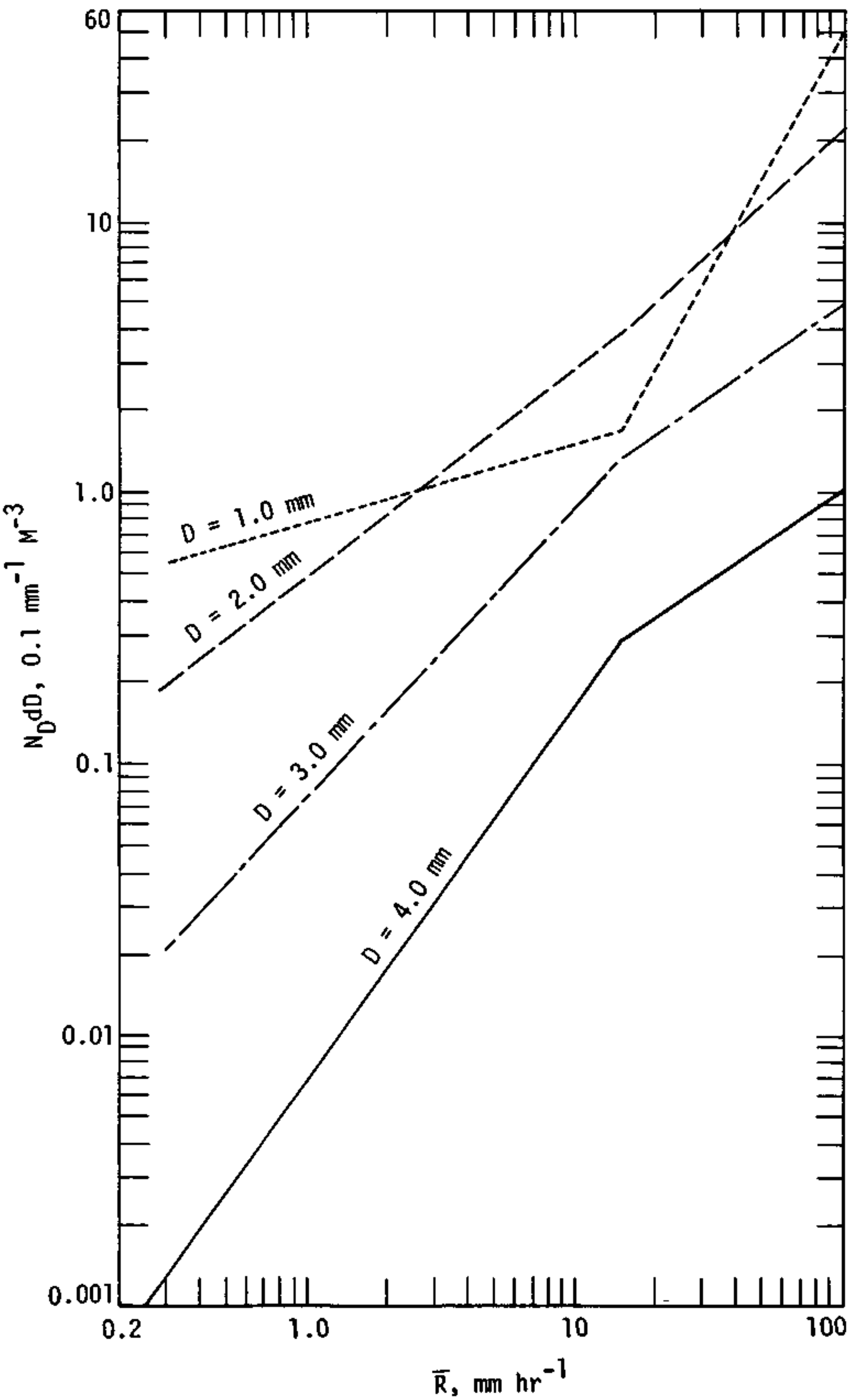
Date	MDT	Conc. type	R max.	Max. D, mm	Hail
7/09	1653-2042	High	1.0	2.8	No
7/10	1439-1455	Low	19.0	5.4	Yes
7/10	1530-1638	High	9.1	3.5	No
7/12	1617-1916	Low	153.6	7.9	Yes
7/13	1825-1837	Low	39.1	4.9	No
7/13	1910-1942	High	35.0	4.8	No
7/13	1957-2041	High	17.6	3.1	No
7/14	1400-2000	Low	2.1	3.0	No
7/15	1602-1714	Low	114.6	4.9	Yes
7/15	1813-1818	Low	18.0	5.0	Yes
7/16	1424-1451	Low	52.0	7.3	No
7/16	1556-1605	Low	13.5	3.2	No
7/17	1733-1743	Low	34.2	4.6	No
7/26	1140-1234	Low	59.9	7.9	Yes
7/27	1418-1524	High	3.1	3.1	No
7/29	1203-1454	High	40.4	6.0	No
7/30	1303-1615	High	3.4	2.9	No
8/01	1540-1650	High	33.9	3.5	No
8/03	1225-1245	Low	92.3	7.9	Yes
8/03	1302-1311	Low	6.9	4.2	No
8/03	1333-1342	High	3.8	2.6	No
8/04	1707-1746	High	2.4	2.6	No
8/06	0013-0023	Low	1.3	2.7	No
8/06	1738-1756	High	14.0	3.2	No
8/06	1909-1930	High	3.5	2.5	No
8/06	2105-2124	Low	8.7	2.7	No
8/07	0819-1112	High	10.8	2.8	No
8/07	1235-1249	High	11.6	4.0	No
8/08	1507-1618	High	14.2	3.0	No
8/09	1253-1304	Low	8.0	3.3	No
8/09	1313-1343	High	25.4	5.9	Yes
8/09	1411-1431	High	1.5	2.3	No
8/10	1406-1414	Low	2.8	3.6	No
8/10	1424-1709	High	2.0	2.7	No
8/14	1543-1602	Low	8.1	3.9	No
8/15	0044-0049	High	5.5	3.4	No
8/15	1306-1328	Low	41.0	7.1	Yes
8/15	1427-1450	Low	29.8	6.0	Yes
8/15	1451-1456	High	6.2	2.9	No
8/15	1457-1525	Low	90.3	7.7	Yes
8/15	1526-1542	High	3.4	3.0	No
8/15	1552-1611	Low	25.6	5.3	No
8/15	1612-1640	Low	6.0	4.1	No
8/16	1541-1550	High	22.7	4.6	No
8/16	1813-1821	High	3.3	2.5	No
8/18	1256-1306	Low	8.7	4.8	Yes
8/18	1350-1401	Low	7.9	5.0	No
8/18	1402-1434	High	12.9	4.1	No
8/19	1530-1605	Low	17.2	4.7	Yes

RAINDROP BREAKUP

The discontinuity in the slope of the hail concentration curve of Fig. 5 occurs near R = 12 mm hr^{-1}. The slope change is toward an increased number of drops per cubic meter at the higher rainfall rates, indicating that there was some change in the drop production mechanism near that rate. This phenomenon was investigated by calculating N_D (the number of drops in a 0.1-mm diameter class) for each rainfall rate at selected diameters from the log-normal distribution curves fitted to the data for the hail concentration showers. The rainfall rates were grouped on an approximately logarithmic scale of 22 groups from 0.1 mm hr^{-1} through the highest rate measured, 154 mm hr^{-1}. N_D-R curves shown in Fig. 6 are comparable to the N_T-R curves of Figs. 4 and 5, but apply to a single 0.1 mm-diameter interval per cubic meter rather than to all drop diameter classes. Fig. 6 explains the origin of the increase in drop production at high rainfall rates as the breakup of drops initially larger than 2.0 mm diameter into drops smaller than 2.0 mm and, necessarily, an increased number of drops.

Fig. 6 also shows that between 2.4 and 40 mm hr~ there are more 2.0 mm drops than 1.0 mm drops. Fig. 7 shows the N_D-R curves for the two extreme sizes of the mode diameter, 1.1 and 1.7 mm. The number of 1.2 mm drops exceeds the number of 1.7 mm drops at R < 3 mm hr~ and R > 35 mm hr .

Figs. 6 and 7 contain an explanation for the decrease in mode diameter at rainfall rates greater than 30 mm hr^{-1} and for



60
10
1.0
0.1
0.01
0.001
0.2
1.0
10
100
$N_D dD$, 0.1 mm^{-1} M^{-3}
$\bar{R}$, mm hr^{-1}
D = 1.0 mm
D = 2.0 mm
D = 3.0 mm
D = 4.0 mm

Fig. 6

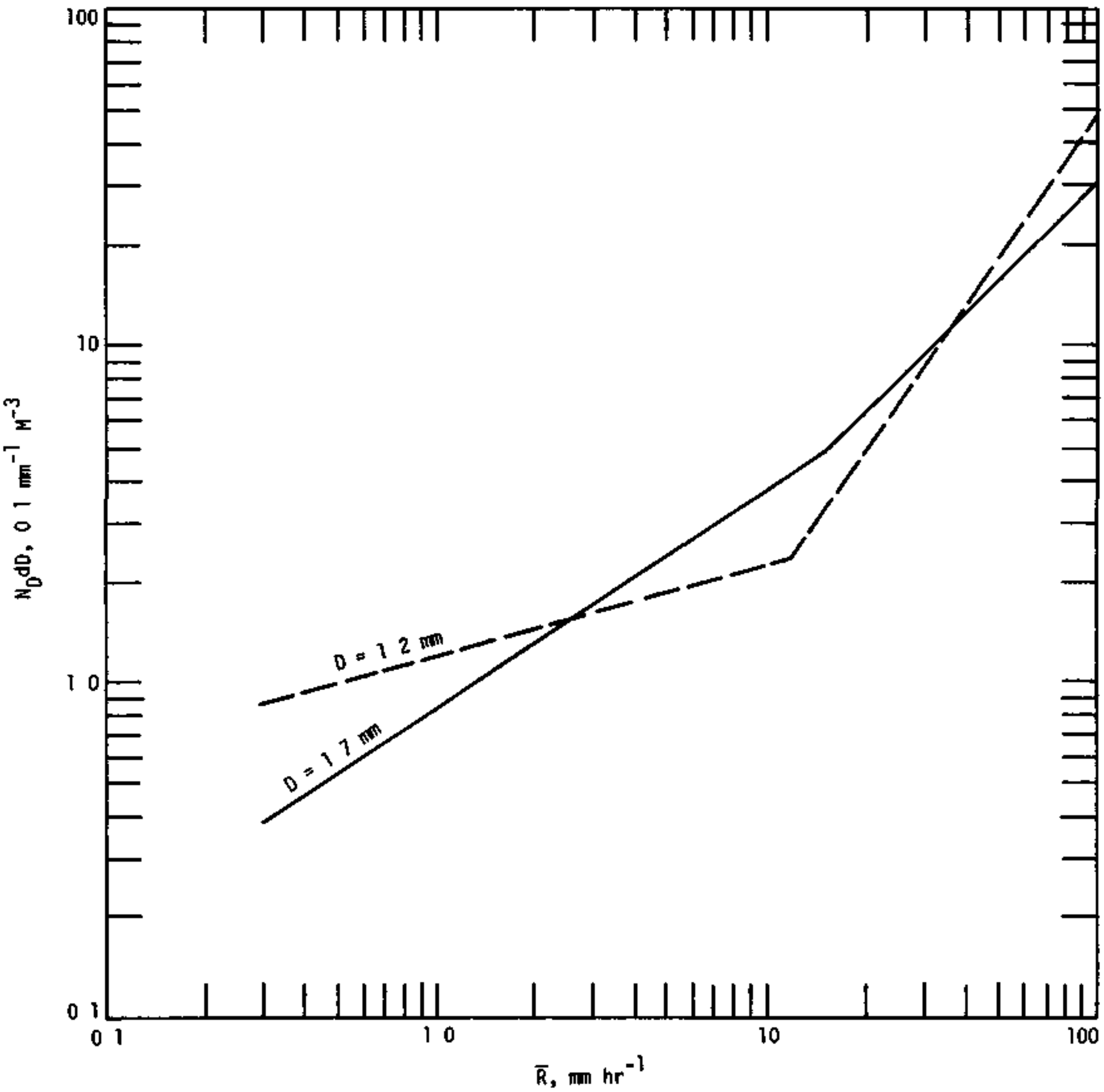


Fig. 7

the increase in the rate of production of raindrops at rainfall rates greater than 12 mm hr^{-1}. The curve for drops of diameter 2.0 mm in Pig. 6 indicates there is very little change in the slope of the curve from the smallest to the largest rainfall rates. Such is not the situation with the 3.0-and 4.0-mm drops. These curves show a definite node at $10 > R > 20$ mm hr^{-1}, the interval in which the number of these drops becomes significant per cubic meter sample. Apparently, the vibrational oscillations due to aerodynamic distortion of drops of $D > 2$ mm is sufficient to cause drop breakup of some drops of those sizes. The number of drops involved in drop breakup is proportional to the number of drops in each diameter interval. Since the slopes of the curves for the 3.0-and 4.0-mm drops in Pig. 6 are the same, it is concluded that the frequency of drop breakup is not proportional to the size of the drop. Lending further credence to this conclusion is the fact that a surprising number of drops with $D > 6$ mm have been photographed in the Flagstaff area and at other camera sites where the higher rates have been experienced.

As the rainfall rate increases, the number of drops available for breakup increases. The exact mode of breakup may take one of several forms (Lane, 1951; Blanchard, 1950; Magarvey and Taylor, 1956). The raindrop camera has fortuitously photographed the beginning of two of them, the bagging and vibrating modes, ir an early version of the camera (Jones and Dean, 1953). Others have speculated that patterns of small drops on filter paper samples indicate that breakup occurred a short distance above the paper.

The mode of breakup determines the number of drops and their sizes that will be produced. The number of small drops produced by breakup begins to outnumber the drops forming the mode of the distribution near D = 2 mm and reduces the mode to some diameter less than 2 mm. The modal diameter change becomes obvious at rainfall rates greater than 30 mm hr^{-1}. However, it will be noted on Pig. 8 that the mode diameter stops increasing near R = 10 mm hr~ and remains relatively constant to R = 30 mm hr^{-1}.

It is of interest to speculate on the number of drops on the average that would be produced were drop breakup forces not operative. If the curve for D = 3.0mm at R< 10 mm hr^{-1} is extrapolated without a change in slope to R = 100 mm hr^{-1}, there would be ten 3.0-mm drops per cubic meter. Similarly, there would be four of the 4-mm drops and three of the 1-mm drops per cubic meter.

Splash Drop Formation

Questions have been raised concerning the housing of the camera and its proximity to the sampling volume as a source for small drops splashing from the housing. The wind was usually less than 1.2 m sec^{-1} at the camera site in Arizona even during the thundershowers. These low wind speeds would make the drift of splashing drops unlikely. Notwithstanding, midway through the collection period, screening was added to the housing nearest the sampling volume and was seen to reduce dramatically the amount of spray observed above the housing during high intensity rains.

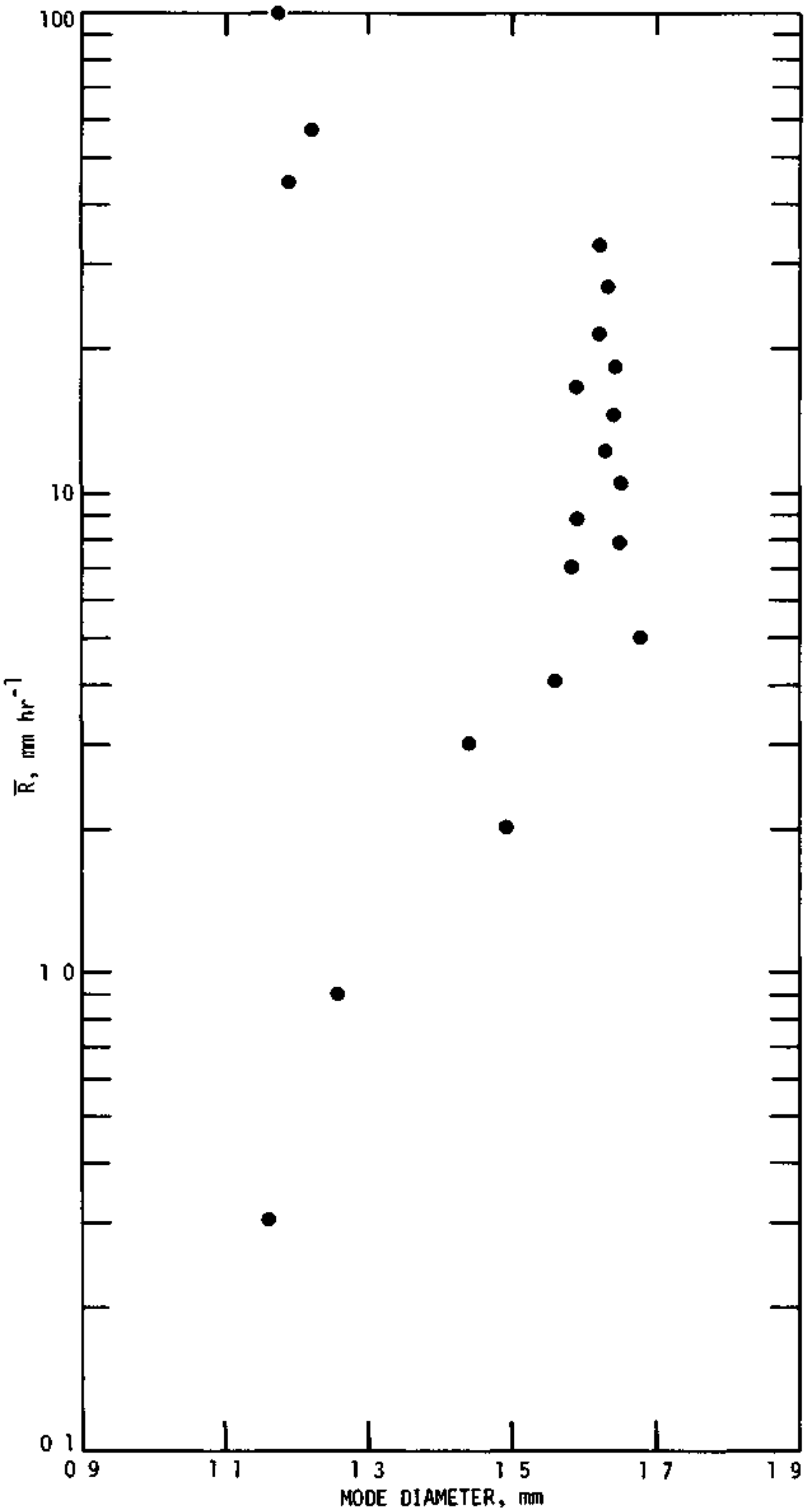


Fig. 8

As a check on the effects of splashing, a 2-group sorting of the low concentration data (which contained most of the high intensity rains) was made to include those showers which occurred before and after the screening was added. The results are shown in Pig. 9. It would be expected that the mode diameter would be smaller for those showers which were photographed before the screening was installed. Just the opposite is seen to be true. However, because of the few samples at the higher rainfall rates, this is not thought to be a real difference. It is concluded, therefore, that the proximity of the camera housing is not responsible for the break in the mode at high rainfall rates.

ORIGIN OP THE FLAGSTAFF DISTRIBUTIONS

Hardy (1963) has discussed the role played by the processes of droplet growth, coalescence, accretion, and evaporation for a specific storm near Flagstaff. He chose the particular storm because it approached the desirable type that remains essentially steady-state for a time allowing the raindrops measured at the surface to leave the cloud in the same size distribution relationship so that only the processes of coalescence and evaporation affect them in their fall to the surface. The rain with which Hardy worked was, of necessity, a high concentration rain falling after the cessation of an earlier thunderstorm. To extend the work of Hardy so that the low concentration showers may be studied requires studying the convective showers when they are not

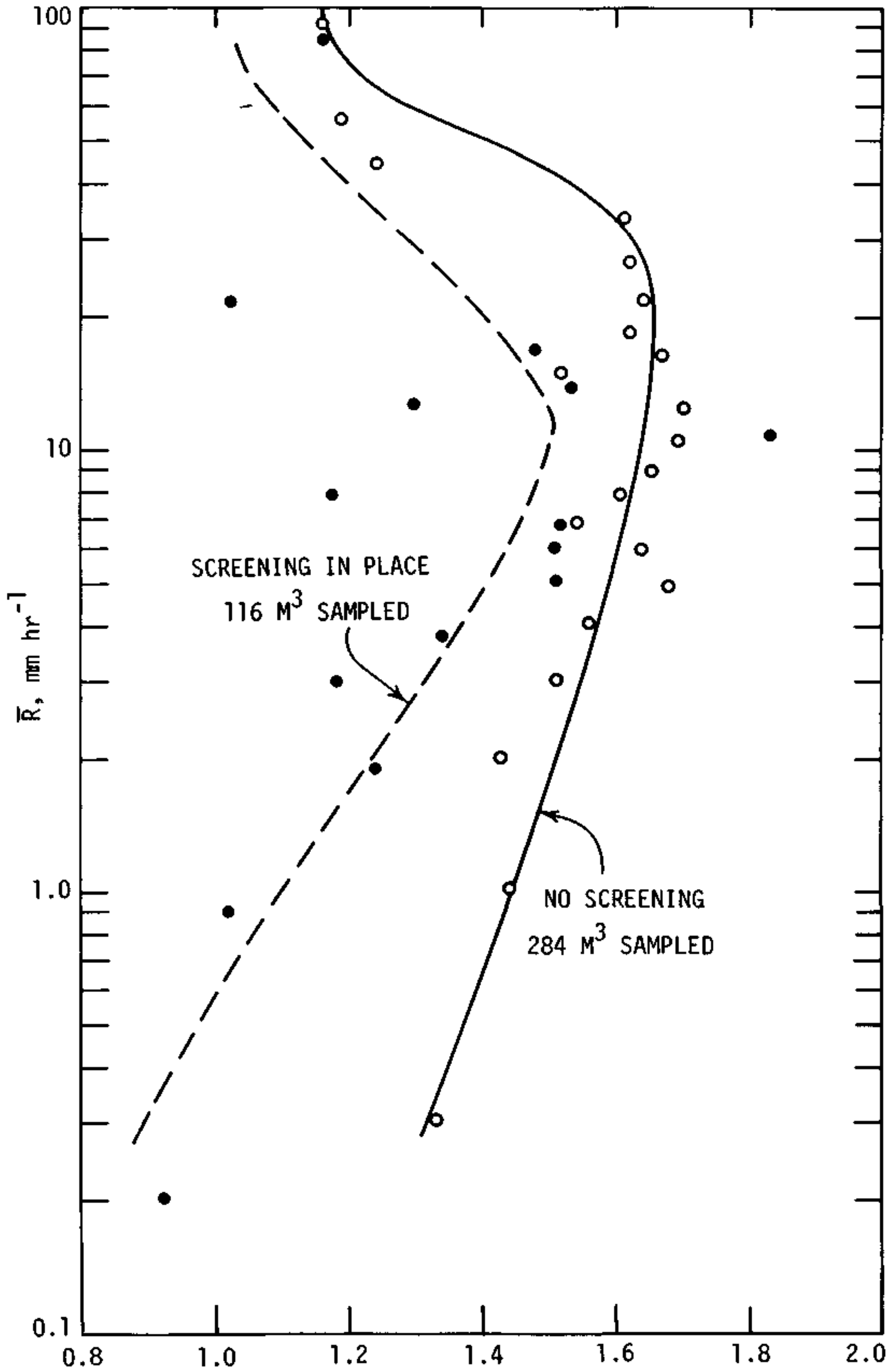

100
10
1.0
0.1
0.8
1.0
1.2
1.4
1.6
1.8
2.0
MODE DIAMETER, mm
$\overline{R}$, mm hr^{-1}
SCREENING IN PLACE
116 M^3 SAMPLED
NO SCREENING
284 M^3 SAMPLED

Fig. 9

steady-state. This may be done by assuming that the drops have maintained their terminal velocity and positional relationships in the fall from the cloud base to the surface measuring site. In addition, coalescence, the capture of one drop by another, has been ignored in the computations for the sake of simplicity. As Hardy has shown, coalescence below cloud base as compared with evaporation in Arizona plays a minor role in the final distribution observed at the surface. It is believed that evaporation is a major influence affecting the drop-size distributions below cloud base in the Flagstaff area. Since the equations for computing evaporation by electronic means may be useful by other investigators, the derivation of the equations and the approximations for the several parameters are given in detail.

Evaporation of Raindrops

The effect of evaporation upon the drops present at the Convective Condensation Level (CCL) has been calculated using the equations given by Kinzer and Gunn (l95l). They give

$$\frac{dM}{dt} = (1 + FD/2s')\left(2\pi DK(\rho_w - \rho_a)\right) \qquad (1)$$

in which K is the diffusion coefficient, M is the mass of the drop, ρ_w is the density of the drop, ρ_a is the density of the environment, s' is the effective thickness of the molecular shell about the drop, and P is a dimensionless quantity measuring the heat or vapor exchange. No theoretical solution is available for Eq. (1), but empirical solutions have been given by Kinzer and Gunn for 1 + FD/2s' and $2\pi DK(\rho_w - \rho_a)$ in the form of tables. Hardy (1963) plotted the tabular values to obtain either equations or tables which could be used in a computer. If $A = 1 + \frac{FD}{2s}$, and $B = 4\pi DK(\rho_w - \rho_a)$, then $\frac{dM}{dt} = AB$. After Hardy, $M = \frac{\pi}{6}D_i^3\rho_w$. Then, $\frac{dM}{dt} = \frac{\rho_v \pi D_i^2}{2}\frac{dD}{dt}$, which may be rewritten as $\frac{dD}{dt} = \frac{2}{\pi\rho_w D_i^2}\frac{dM}{dt}$.

If the drop is falling at its terminal velocity, V_i, (Gunn and Kinzer, 1949; Battan, 1964) then $\frac{dD}{dt} = \frac{dD}{dZ}\frac{DZ}{dt} = V\frac{dD}{dZ}$ in which Z is the height coordinate. When combined in differential form, the equation becomes

$$dD = \frac{2AB}{\pi\rho_2 D^2 V_i}\, dZ, \qquad (2)$$

which is the form used by Hardy.

It would be desirable to enter the equation in pressure coordinates directly from values picked from a sounding plot on an aerodynamic diagram rather than height coordinates. This may be done through the hydrostatic equation, $dZ = -\frac{dP}{\rho_a g}$. P is the atmospheric pressure. ρ_w may be neglected as being practically unity, leaving the final equation as

$$dD = -\frac{2AB}{\pi\rho_a D_i^2 V_i}\, dP. \qquad (3)$$

The tabular values for A may be fitted to a curve such that $A = L(l - 0.00264 \times T)$. The coefficient L is determined from one of two equations depending upon the drop size. If $D \leq 2.0$ mm, then $L = 2.90(10D_i)^{1.49}$, if $D > 2.0$ mm, then $L = 1.80(10D_i)^{2.20}$.

The parameter B is found from $B = k(1.0 - e_a/e_s)10^{-6}$. Again, two equations are necessary to define the parameter, k. If $T \leq 10C$, $k = 0.77 + 0.023T$ and if $T > 10C$, $k = 0.425 + 0.0575T$. The vapor pressure, e , is calculated from an equation from

Kiefer (l941), $1n(e_a/6.105) = 25.21(T_d/(T_d + 273)) - 5.31 \times 1n((T_d = 273)/273)$, 1n which T_d is the temperature of the dew poind obtained from the sounding. The saturation vapor pressure is calculated from the same equation by using the actual temperature, T, in place of T_d.

Tne atmospheric density, ρ_a, is calculated from $\rho_a = (l0^{-6} \times P)/(2.872 \times T')$. P is the atmospheric pressure in millibars from the sounding and the virtual temperature, $T' =(273 + T + 0.622 \times e_a)/(6 \times P)$.

The dependence of the concentration on the evaporative power of the atmosphere in which the clouds formed may be obtained by 1) determining the resultant diameter of a drop falling from the CCL to the surface and 2) comparing this diameter with the concentration of drops at a normalized rainfall rate of 1.0 mm per hour. Accordingly, the resultant diameter of an initial 1.2-mm drop was determined for each day on which a sounding was made at the Navajo Army Depot, and these were plotted against A as shown in Pig. 10. There appears to be no correlation for the 15 storms. However, if the correlation is checked for those soundings released within 2 hours of the storm time at Fort Valley, a correlation coefficient of O.83 is obtained. These data points are indicated by the open circles in Fig. 10. This good correlation between the number of drops of diameter 1.2 mm and the drying power of the air mass is assumed to be a result of the stagnation of the air mass surrounding the sampling site and its subsequent modification by the thunderstorms within it.

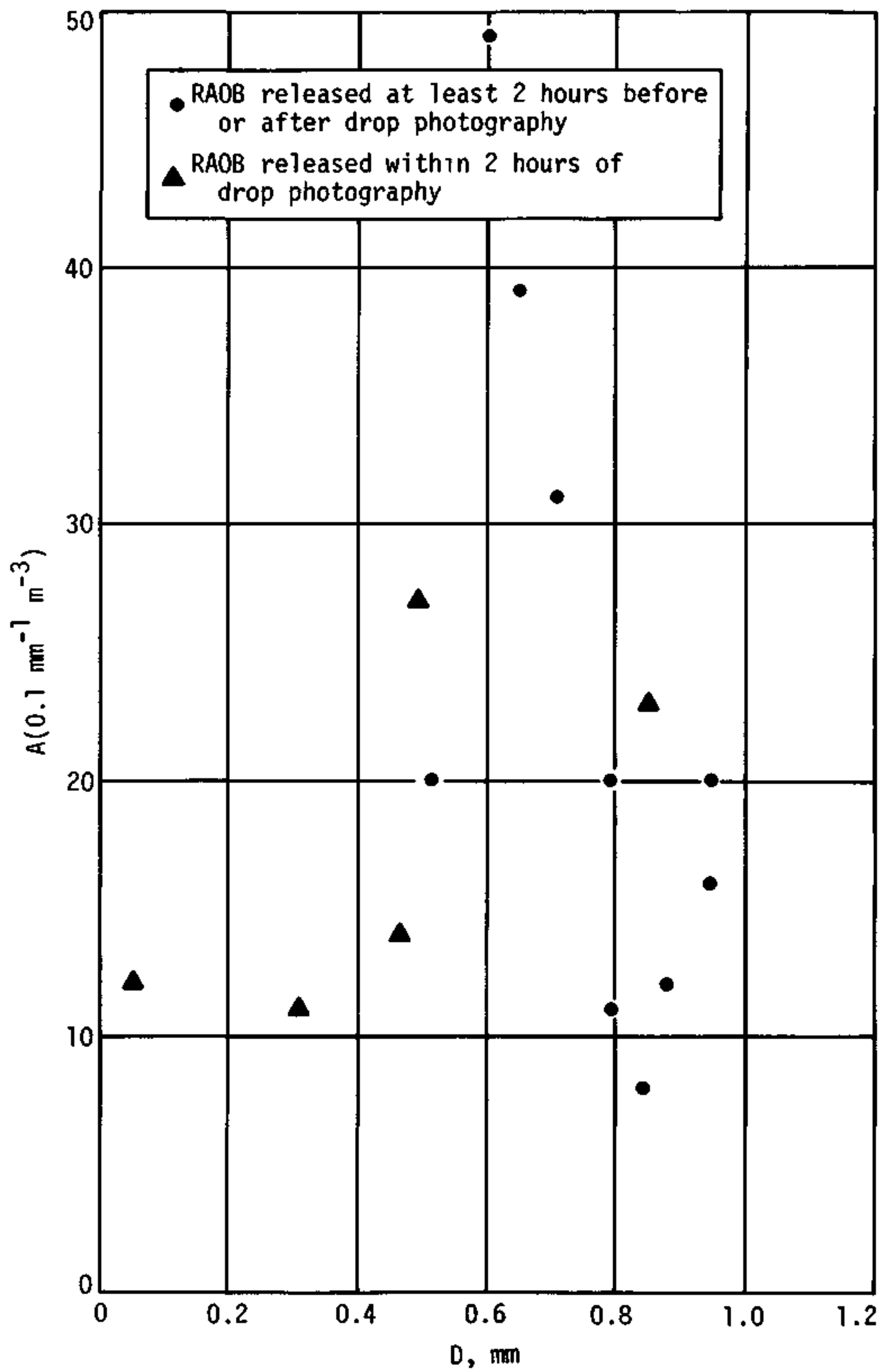

RAOB released at least 2 hours before or after drop photography
RAOB released within 2 hours of drop photography
A(0.1 mm^{-1} m^{-3})
D, mm
0
10
20
30
40
50
0.2
0.4
0.6
0.8
1.0
1.2

Fig. 10

Modification of Air Mass

Pig. 11 is a skew T - log P plot of the soundings released at 0716 MDT and 1245 MDT of 9 August 1967. These soundings illustrate the modification of the air mass by the release of energy by the thunderstorms active in it. In Fig. 11 the early morning sounding at O716 MDT shows the radiation inversion close to the surface usually found after a clear night on the desert. This inversion is destroyed by surface heating from the sun by 1245 MDT. In addition, warming of the lower atmosphere has occurred from the surface to 500 mb and the moisture content within this layer has increased, particularly between 600 and 500 mb, in response to the thunderstorms active between the soundings (Braham, 1952). Warming has also occurred between 420 mb and 300 mb as the moisture increased in that layer, probably indicating the presence of cirrus densus from the earlier thunderstorms.

Cloud Base Drop-Size Distributions

Convective showers may be studied for their distributions at cloud base if the assumption is made that vertical shear is negligible and the arrival time at the surface for the raindrops is a function only of the terminal velocity of the drops. Thus, drops measured at a known time at the surface may be hypothetically lifted to reconstitute the distribution as they left the cloud base (CCL). Accordingly, the drop sizes for two storms have been rearranged in time by their fall times from cloud base, and the drops which left the cloud base together were grouped to form a

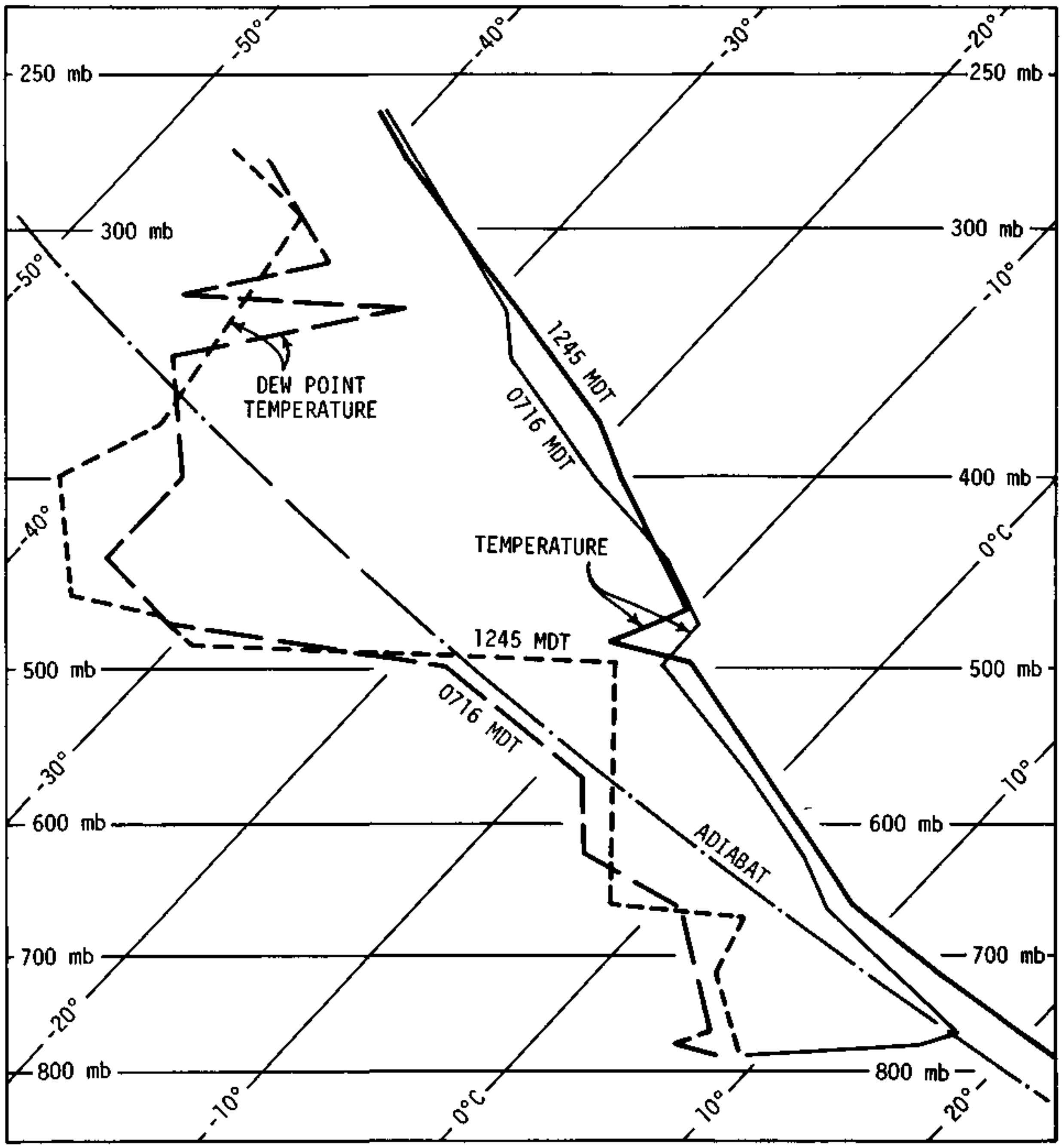
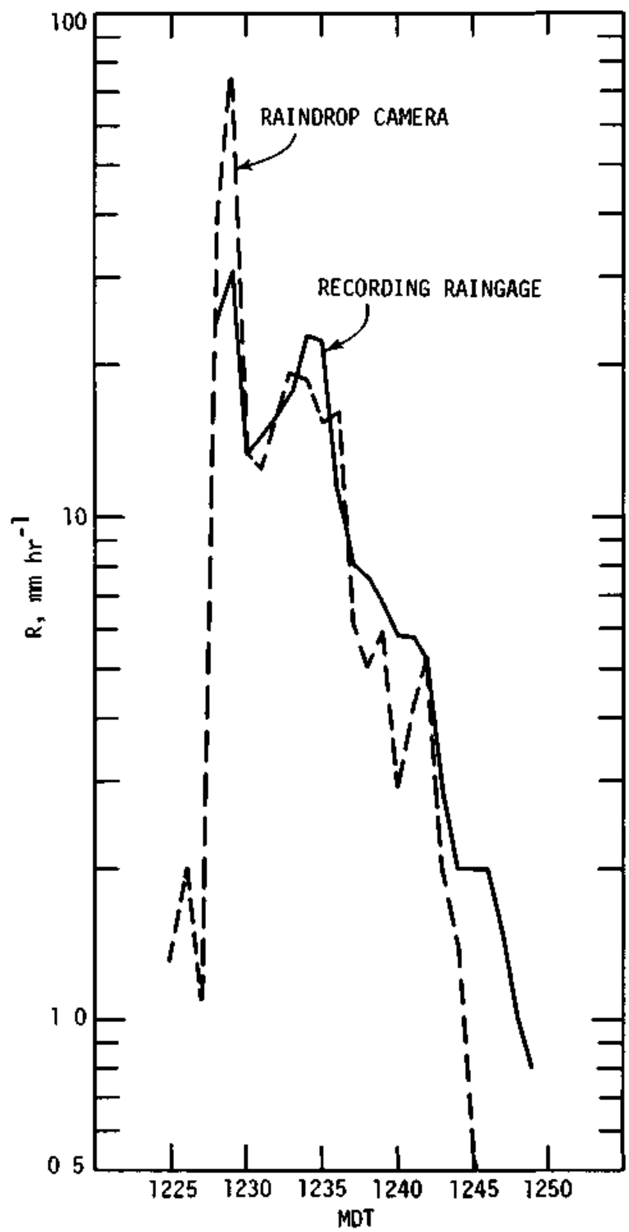

250 mb
250 mb
300 mb
300 mb
400 mb
500 mb
500 mb
600 mb
600 mb
700 mb
700 mb
800 mb
800 mb
-50°
-40°
-30°
-20°
-10°
0°C
10°
20°
DEW POINT
TEMPERATURE
TEMPERATURE
1245 MDT
0716 MDT
1245 MDT
0716 MDT
ADIABAT

Fig. 11

new 1-minute sample. The two storms studied were both thunderstorms and neither was artificially seeded.

The Low Concentration Thunderstorm of 8/3/67. This was the first storm of the day at the camera. The record from the M-33, S-band radar of the Bureau of Reclamation at the Flagstaff Airport indicated that this storm remained essentially stationary with the movement apparent at the sampling site being one of growth and decay. Echoes formed on the west flank of the San Francisco Peaks and just north of A-1 Mountain between 1205 and 1210 MDT. These echoes grew to cover the camera site when the camera began operation at 1225 MDT. The camera continued to photograph through 1245 MDT. The camera record by integration of the indicated rainfall rates showed that 4.3 mm of precipitation fell while the nearby raingage indicated 3.6 mm. A comparison of the instantaneous rainfall rates read for each minute from each instrument is shown in Fig. 12. The camera indicated a measurable rainfall before the raingage and also indicated a larger rainfall rate than the raingage. This is because of the time required to wet the raingage collector and establish flow over a finite distance into the weighing bucket. This hydraulic lag tends to smooth the rainfall rates detected by the gage. Total rainfall as measured bv the network of recording raingages is shown on the isohyetal map, Fig. 13.

After the drops were raised to the CCL, they were regrouped into 0.1-mm class intervals centered upon 0.1 mm; i.e., D equals the diameter of all drops from D - 0.05 mm through D + 0.05 mm.



100
10
1 0
0 5
R, mm hr-1
RAINDROP CAMERA
RECORDING RAINGAGE
1225
1230
1235
1240
1245
1250
MDT

Fig. 12

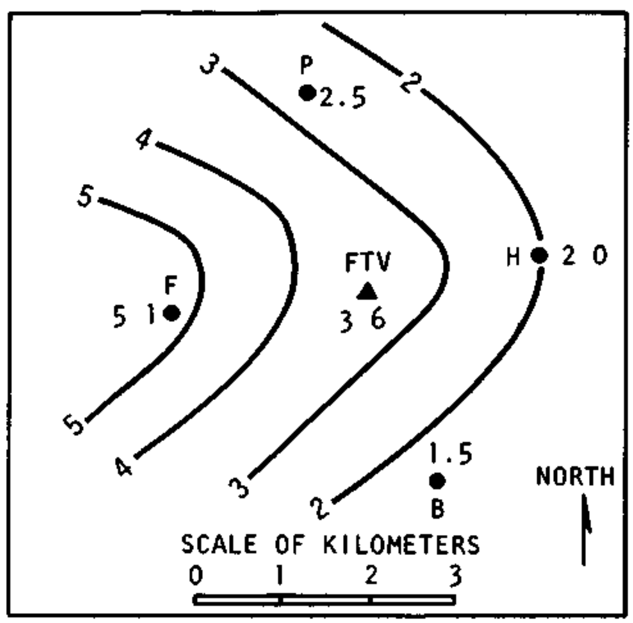

P
2.5
2
3
4
5
FTV
3 6
H
2 0
F
5 1
5
4
3
2
1.5
B
NORTH
SCALE OF KILOMETERS
0 1 2 3

Fig. 13

Obviously, there will be no drops smaller than the smallest drop measured at the ground, and, in general, there will be no drops as small as the smallest drop measured at the ground since the greatest change in diameter is at the smallest sizes.

Table 5 lists the radiosonde values for the sounding released at 1245 MDT on 8/3/67. This sounding was used as the sounding for the study of the low concentration storm beginning at the camera at 1225 MDT on 8/3/67.

The surface pressure at the camera site was usually 783 mb, whereas the pressure at the sounding release point was always near 788 mb. The CCL was near 600 mb from this sounding.

Table 5. Sounding for 8/3/67, 1245 MDT

Pressure (mb)	Temperature (°C)	Dew Point (°C)
600	3.3	-2.5
610	4.2	-1.7
620	5.1	-1.4
630	6.0	-1.0
640	7.0	-0.7
650	7.8	-0.3
660	8.7	0.1
670	9.6	0.3
680	10.9	1.1
690	12.2	2.0
700	13.6	3.0
710	15.0	4.0
720	16.3	5.0
730	17.7	6.0
740	19.0	6.9
750	20.2	7.8
760	21.6	8.6
770	23.2	9.5
780	24.9	10.3
790	26.0	11.1

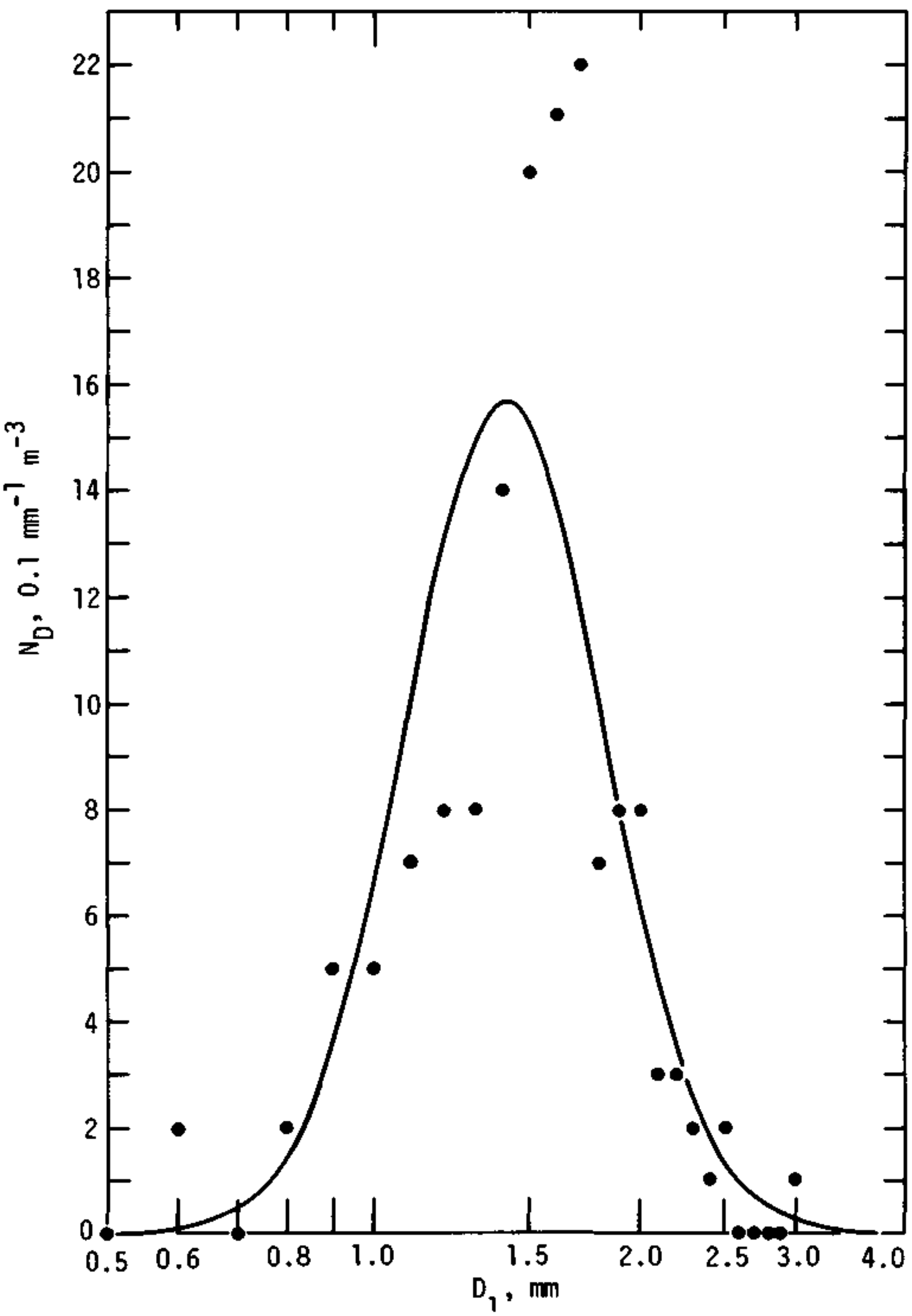
It was found that the drops could be redistributed by their average fall velocity (700-mb velocity). Thus, the drops arriving at the surface at 1228 MDT with sizes including 1.0 and 1.1 mm were grouped with the drops from 1.2 to 1.6 mm for 1227, with the drops from 1.6 to 2.2 mm for 1226, and with the drops from 2.2 mm through the largest size measured for 1225 MDT. The distribution was then called the observation for 1223 MDT. This procedure was performed for all observations through 1242 MDT since there could be no drops smaller than 2.1 mm for observations beyond 1242 MDT. This procedure was performed before the drops were raised to the CCL.

The log-normal distribution equation has been found to fit the distributions measured at the surface by many investigators (Mueller, 1967). An example of the initial distribution as observed at 1239 MDT of 8/7/67 is shown in Pig. 14. Also shown in Fig. 14 is the curve of the distribution as fitted to the log-normal form. The log-normal equation is

$$N_D dD = N_T/D\sqrt{2\pi}\ \ln\ \sigma\left[\exp - 1/2\left(\ln(D/D_G)/\ln\sigma\right)^2\right]dD$$

in which σ is the standard deviation and D_G is the geometric width.

Pig. 15 depicts the evolution of the drop-size distribution at the CCL as based upon the observations at the surface with the effects of evaporation taken into account. There is a rise and fall in the amplitude of the mode with the peak occurring at the base of the cloud at 1225 MDT. The smallest drops falling to the ground were 1.2 mm or larger when they left the base of



22
20
18
16
14
12
10
8
6
4
2
0
N_D, 0.1 mm^{-1} m^{-3}
0.5
0.6
0.8
1.0
1.5
2.0
2.5
3.0
4.0
D_1, mm

Fig. 14

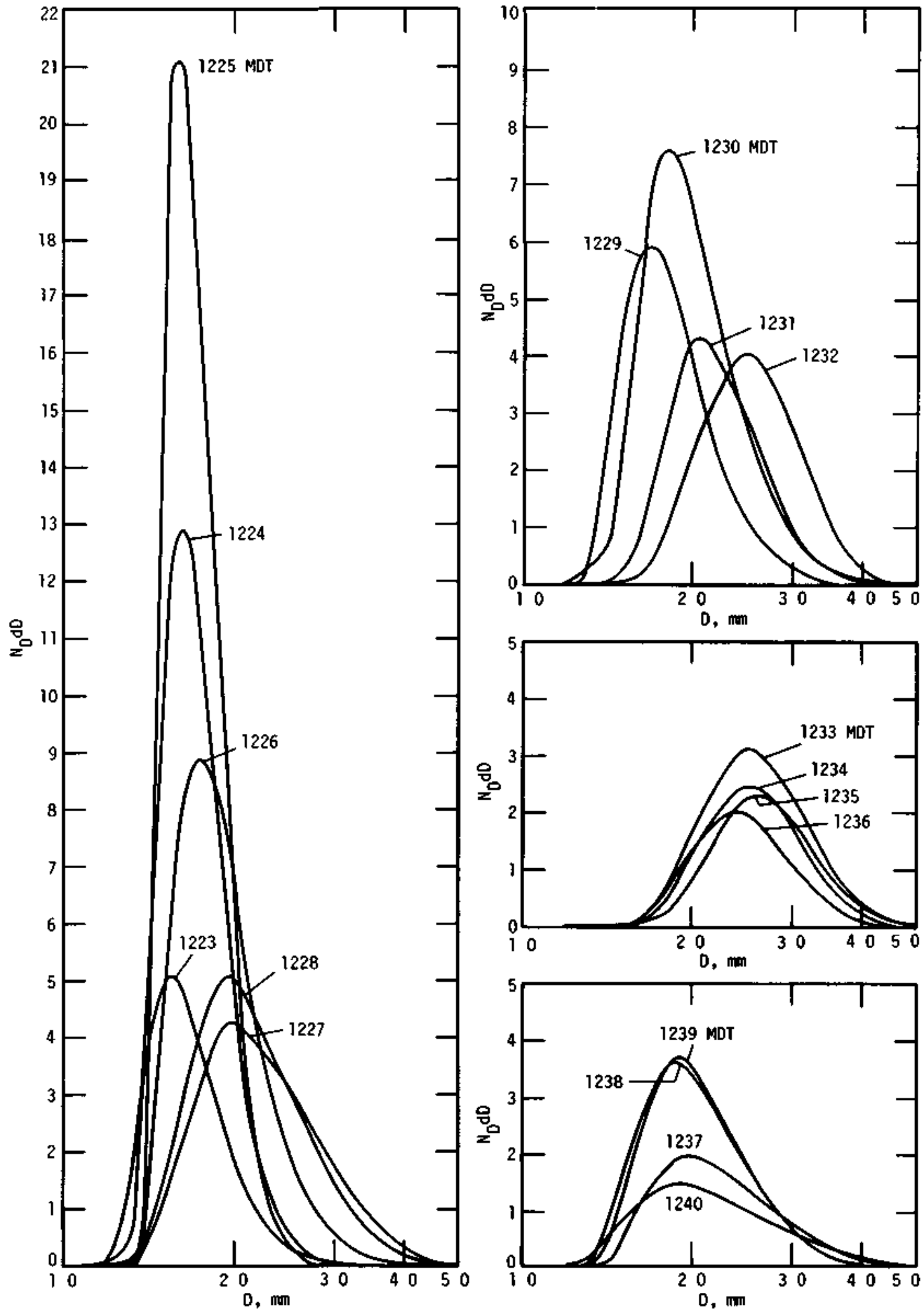

1225 MDT
1224
1226
1223
1228
1227
$N_D dD$
D, mm
1230 MDT
1229
1231
1232
1233 MDT
1234
1235
1236
1239 MDT
1238
1237
1240

Fig. 15

the cloud, since evaporation eliminates all drops smaller than 1.2 mm diameter in the fall to the surface. Thus, the smallest drop depicted at the base of the cloud was 1.2 mm in diameter. It is obvious that smaller drops existed at cloud base, but these drops evaporated to increase the humidity of the downdraft below the cloud. A very crude estimate of the liquid water evaporated in the downdraft is 1.2×10^{12} grams. This estimate assumes that the average relative humidity of 95% at the surface during the storm is representative of the total downdraft, and the downdraft has a volume defined by the depth from the CCL to the surface of 2.1×10^{2} m and an area of 1.1×10^{10} m^2. The average relative humidity within the column below cloud base was found to be 50% at an average temperature of 13.6 C. These figures result in an average saturation mixing ratio of 14.4 g/kg for the column with a deficit of 7.2 g/kg or 6.1 g/m^3. It would be desirable to be able to compute the number of drops that evaporated in each class interval in order to obtain this mass of water substance. Knowledge of the initial distribution is necessary for this exercise and, since the initial distribution is the desired product, the calculation may not be performed.

It had been anticipated that the reconstitution of the surface distributions to those at the CCL would reveal a distribution defined by $N_D dD = N_o \exp(-\Lambda D) dD$ (Marshall and Palmer, 1948), wherein $N_D dD$ is the number of drops per cubic meter of diameter between D and D + dD mm, N_o is the number of drops at D = 0, and Λ is an empirical variate related to the rainfall rate. Intuitively, it might be assumed that there would be a continuum of drop sizes at the CCL from the micron sizes of cloud droplets to the largest sizes measured. This has not been found to be true for the thunderstorm of 8/3/67 even though, as a precautionary measure, the calculations were confined to those in which complete confidence was held for their detection and measurement ($D \geq 1.0$ mm).

Hardy (1963) and Braham (1952) have shown that evaporation within the downdraft begins with the formation of the downdraft at a much higher level than the CCL within the cloud. This evaporation must reduce the number of small raindrops surviving the fall to the CCL and completely eliminate the cloud droplets within the downdraft at the CCL.

The evaporation of the drops as they fall in the column of air beneath the cloud modifies this air by the extraction of heat and the addition of moisture. The first drops to fall from the cloud will fall into unmodified air whereas those drops falling later in the storm will be falling into air with less evaporative power and will be subject to less evaporation because of it. The sounding made at Navajo Army Depot concurrent with the drop-size measurements was released in rain-free air. Thus, the air represented by the sounding had not undergone modification by evaporating raindrops and does not represent the environment through which the drops falling late in the storm would pass. This change in evaporative power of the air mass has been ignored in the calculations of drop evaporation.

The High Concentration Thunderstorm of 8/7/67. This thunderstorm moved from the south at a speed of 4.5 m sec^{-1} toward the camera site as the last of a series of showers for the day.

The echo had been in existence for at least 35 minutes when rain began at the camera site. Since the average cell life of a thunderstorm is approximately 35 minutes, this was very likely a multicellular storm with succeeding cells seeded from above by ice crystals from the older cloud towers. The radar record also shows that this was a multicellular storm. The storm moved from the general direction of the radiosonde release point. The rain began at the camera at 1235 MDT and continued until 1250 MDT. The radiosonde was released at 1252 MDT. The proximity of the sounding both in time and space makes it very likely that the air mass through which the sonde passed was representative of the air mass in which the showers were occurring. Table 6 lists the 1252 MDT sounding between the surface and the CCL.

Table 6. Atmospheric Sounding for 8/7/67, 1252 MDT

Pressure (mb)	Temperature (°C)	Dew Point (°C)
650	6.6	2.1
660	7.3	3.1
670	8.0	4.0
680	8.7	5.0
690	9.3	6.0
700	10.0	7.0
710	10.7	7.9
720	11.5	8.8
730	12.3	9.7
740	13.1	10.6
750	13.9	11.4
760	14.7	12.3
770	15.4	13.2
780	16.2	14.0
790	16.8	14.7

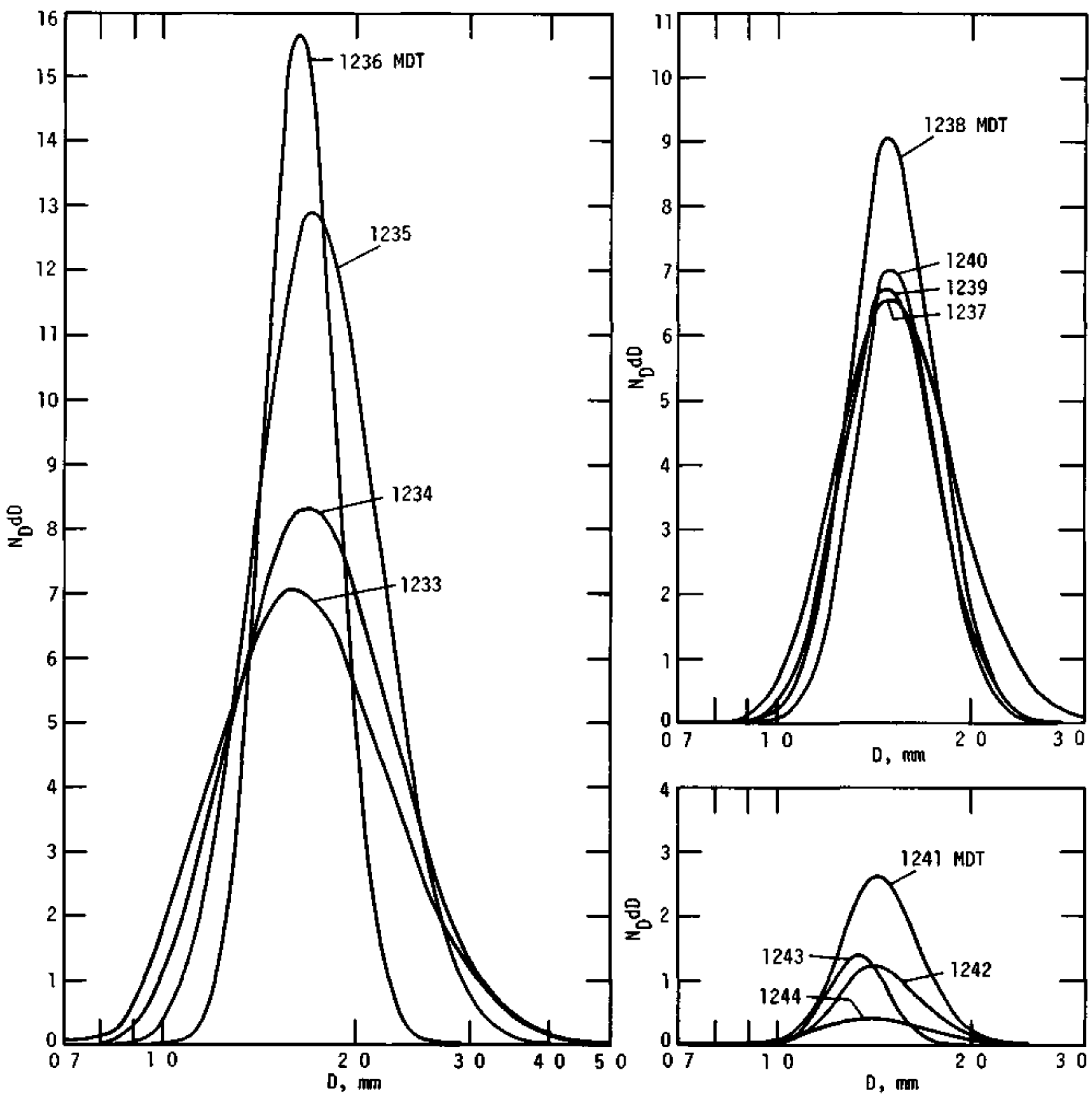
Reference to Table 5 shows the difference between the sounding for the low concentration storm of 8/3/67 and the sounding of this high concentration storm. The 700-mb level temperatures and dew point temperatures for the soundings are particularly noteworthy. The low concentration sounding has a temperature of 13.6 C and a dew point spread of 10.6 C whereas the high concentration sounding has a temperature of 10.0 C and a dew point spread of 3.0 C, illustrating the much drier air mass with the low concentration shower.

The sounding released at 0700 MDT on 8/7/67 (not shown) was saturated from the surface to 507 mb except for minor drying near 605 mb. By the time of the second sounding at 1252 MDT, there was some drying in the lower levels although the air was still relatively moist as noted above. The later sounding clearly shows the transport of water vapor aloft as the storms continued through the morning. The moisture increased from 0.2 g/kg to 0.8 g/kg at 372 mb between the 0700 MDT sounding and the 1252 MDT sounding. This agrees with the findings of Braham (1952) noted earlier. For this storm of 8/7/67, the peak in moisture increase was near 7,600 m MSL at a temperature of -20 C. By contrast, the peak in moisture increase aloft for the storms of 8/3/67 occurred at 9,800 m. This would indicate that the storms of 8/3/67 reached greater heights than did the storms of 8/7/67.

The raindrop data for 8/7/67 have many more drops in sizes between 0.5 and 2.0 mm than do the data for 8/3/67. Correspondingly, the largest drop measured was 4.0 mm in diameter even though the rainfall rate reached 12 mm/hr. At that rate on 8/3/67, raindrops as large as 5.5 mm were measured. Pig. 16 depicts the drop-size distributions as they existed at the CCL on 8/7/67. By comparison with the distributions of 8/3/67, it is obvious that much smaller drops (as small as 0.7 mm diameter) reached the surface on 8/7/67 than on 8/3/67 when the smallest drop to reach the surface from the CCL was 1.2 mm diameter at the CCL. In spite of the greater number of small drops on 8/7/67, the distribution at the CCL is still closely approximated by the log-normal distribution rather than the Marshall-Palmer type. The distributions on 8/7/67 are much narrower than the distribution for the low concentration case. The 8/7/67 distributions are broader in the early portion of the storm and narrow sharply by the fourth minute as the number of drops in the mode increase. This storm moved over the camera with the highest rate at the beginning of data collection. This was shown by both the camera and the trace from the nearby raingage. The greatest amount of rainfall and the most intense rate occurred at the Forest (P) recording gage to the west of the camera. No hail occurred at any of the hail detectors within 2.4 km of the camera during this storm.

CONCLUSIONS

Of 25 low concentration storms, 11 were accompanied by hail. Of 24 high concentration storms, 1 was accompanied by hail. Since the concentrations were defined by the occurrence of hail, these



1236 MDT
1235
1234
1233
N_DdD
D, mm
1238 MDT
1240
1239
1237
1241 MDT
1243
1242
1244

Fig. 16

statistics are not surprising. However, hail did not occur with a seeded storm within the observing network nor elsewhere to the author's knowledge. This points to the inhibition of the formation of hail by the artificial glaciation of the storm clouds. The seeded clouds were all found to be of the high concentration type. This statistic may be a fortuitous result of seeding in moist air masses only; but it is more likely that the seeding results in high concentrations.

The largest drop measured among the high concentration showers was of 6.0 mm diameter which is the usually assumed size limit for raindrops. Since the largest drop measured in low concentration showers was of 7.3 mm diameter and in showers with hail was of 7.9 mm diameter, it is very likely that the larger drops accompanying the low concentration showers and showers with hail are possible because of the hail. The presence of either a small particle of ice in the water drop or an air bubble remaining from the melted hailstone stabilizes the oversized drop to prevent breakup or may permit it to grow by coalescence without breakup.

The peculiarity of the low concentrations of drops in some Flagstaff showers is thus explained by hail having been present in those showers and at least the remnants of the hail being contained in the drops larger than D = 6.0 mm. The larger drops experience less loss of mass and diameter than the smaller drops contained in the same storm. In fact, the smaller drops experience sufficient evaporation so that they become an unmeasurable fraction of the number of drops to be measured within 1 m^3.

The evaporative power of the air mass controls the mechanism which is responsible for presence or lack of hail formation in the shower. The cooling of evaporation is available for the removal of the heat released during the freezing of the drops and the subsequent growth of the embryo hail through the wet stage. Thus, the low concentration showers observed near Flagstaff are the result of a relatively dry air mass and high cloud bases. Such conditions exist in dry climates elsewhere, and low concentration showers should occur in those climates.

The evidence of drop breakup and the sizes that the process affects should have applications in future theoretical studies of the processes responsible for the drop-size distributions observed at the earth's surface. Certainly, the breakup process will take its place in importance with the processes of accretion, coagulation, coalescence, and evaporation.

ACKNOWLEDGMENTS

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